Artificial Neural Networks-Assisted Geometric Shaping Optimization Including Gray-like Mapping

Xiang Liu^{1,2}, Jiao Zhang^{1,2*}, Min Zhu^{1,2*}, Bingchang Hua², Yucong Zou^{1,2}, Qinru Li¹, Yingxin Wei¹, Weidong Tong¹, Yuancheng Cai^{1,2}, Mingzheng Lei², Liang Tian², and Aijie Li²

¹ National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China ²Purple Mountain Laboratories, Nanjing, 211111, China <u>*jiaozhang@seu.edu.cn; minzhu@seu.edu.cn</u>

Abstract: Artificial Neural networks-based geometric shaping is proposed that includes Gray-like mappings. Over 0.2dB gain in *GMI* and *BER* improvement is achieved over a wide range of SNRs without requiring any presumed model for the channel.

1. Introduction

The pursuit of capacity-approaching modulation formats is under intensive research to close the 1.53dB Shannon limit. Constellation shaping is now a well-established technique to boost the transmission capacity and operate close to the theoretical achievable information rate (*AIR*). The GS provides lower shaping gains than probabilistic amplitude shaping (PAS) since constellation lack Gray-like code, but the implementation complexity and difficulty are low. In existing works, the pairwise optimization (PO) algorithm minimizing the bit error rate (*BER*) is used to optimize *N*-dimensional constellation [2]. However, PO algorithm is unstable, softly fall into the locally optimal solution. The end-to-end learning approaches need differentiable channel model to ensure backpropagation through the whole system to optimize, and easily find locally optimal solution when randomly initialized [3].

It is well-known that, the *GMI* can be maximized when the QAM constellation follows Maxwell-Boltzmann (MB) distribution for an additive white Gaussian noise (AWGN) channel [4]. While the approximation for communication channel is valid for a wide range of channels, there are situations that this approximation could be inaccurate such for the cases that nonlinearity of the optical fiber is dominant [5]. A laudable goal could be finding the proper position of the constellation without requiring any presumed model for the channel.

In this paper, we use a simple, yet effective artificial neural networks (ANNs) models to learn geometric constellation avoiding falling into locally optimal solution including Gray-like mapping optimization, such that channel model characteristics are captured by the model. It makes no assumption on the channel model and is easily scalable to constellations of higher order and higher dimension.

2. GS-ANNs optimization model

The key idea of GS-ANNs optimization model is an unsupervised learning method embedded a channel model with two neural networks (an encoder and a decoder) to reproduce the input binary stream at the output by minimizing the cross-entropy loss, as shown in Fig. 1.



Fig. 1. Block diagram of the GS-ANNs optimization model

2.1. ANNs-based Communication Systems

It is bound to learn a representation robust to the channel impairments. An ANNs-based optimization model with encoder and decoder is mathematically described as follows

T3B.5

$$\begin{aligned} x &= f_{\theta_f}(s), \\ y &= C_{Gauss/NNs}(x), \\ r &= g_{\theta_u}(y), \end{aligned}$$
(1)

where $f_{\theta_{f}}(\cdot)$ is the encoder, $g_{\theta_{s}}(\cdot)$ is the decoder and $C_{Gauss/NNs}(\cdot)$ is the Gaussian metric or Neural Network metric channel model. The goal is to reproduce the input *s* at the output *r* through the latent variable *x*. The weights and biases of the encoder and decoder NNs are represented by θ_{f} and θ_{g} , respectively. The model parameter θ are trained by minimizing the cross-entropy loss

$$L(\theta) = \underbrace{\frac{1}{K} \sum_{k=1}^{K} \left[-\sum_{i=1}^{M} s_i^{(k)} \log\left(r_i^{(k)}\right) \right]}_{Expectation}.$$
(2)

where K is the training batch size. The average of the cross-entropy obtained is then back propagated to optimize the NNs weights. The size of the neural networks, number of layer and hidden units, are chosen depending on the order of the constellation.

2.2. Constellation Shaping Optimization

The goal is to maximize the generalized mutual information (*GMI*) by optimizing the location of the constellation points including Gray-like mapping without requiring any tractable model of the channel. A message s chosen from a set of M possible message $\{1, 2, ..., M\}$ is trained with so called one-hot vectors. The dimension of input and output space is equal to the order of the constellation, and the dimension of the latent space is equal to the dimension of the constellation. At the transmitter, binary vectors $s = (s_1, ..., s_m) \in \{0,1\}^m$ are fed into a symbol modulator which maps each symbol s into a constellation point x according to $x = f_{\theta f}(s)$, where θ are the NN parameters (i.e., weights and biases). The normalization before the channel poses an average power constraint to ensure the power efficiency of the resulting constellation, the normalized signals presented with I_N and Q_N in Fig.1 pass through the real-time channel. The received symbol y is passed through a decoder NN with trainable parameters θ_g , which maps each symbol y to a probability vector over the set of symbols. The mapping defined by the demodulator is denoted by $p_{\theta_g}(s | y)$, an approximation of the true posterior distribution.

3. Simulation results

The *GMI* achieved by NNs is compared to start-of-the-art modulation schemes considering AWGN channels. Training is performed with the Adam stochastic gradient descent (SGD), and learning rates is set as 10^{-3} . We show the results obtained when both the locations and Gray-like mapping of the constellation points are optimized.

The training set size determines the accuracy of the gradient, it is set in multiples of M. In Fig.2(a), the convergence of the mean loss is shown for different training size. A larger training size leads to faster convergence and better final performance. Fig.2(b) illustrates the same characteristics by describing the mean *BER* performance at different training sizes. The constellations are evaluated using the SNR and *GMI*, which constitutes the AIR under an auxiliary channel assumption. We basically assume that the decoder neural network has learned a probability distribution of the channel as auxiliary channel within the receiver. Fig.2(c) shows the NNs metric value is approaching to the Gaussian *GMI* metric, it verifies the output of the neural network with sigmod activation can serve as an *LLR* estimation. Thus, we can use the NNs model to optimize GS and mapping by learning the probability distribution of the channel.



T3B.5

Fig. 2. (a) Convergence of mean cross entropy loss for different batch sizes; (b) Convergence of mean BER for different batch sizes; (c) GMI performance comparison for Gaussian metric and NNs metric.



Fig. 3. (a) Geometric constellation optimization of 64QAM including Gray-like mapping; (b) Mean BER comparison against SNR; (c) GMI performance comparison against SNR.

Geometric constellation shape of 64QAM is considered to optimize the position and Gray-like mapping, other formats can also be extended. In Fig. 3(a), an example constellation is shown, illustrating that a Gray-like mapping is achieved. It clearly demonstrates the constellation points with Gray-like optimized by NNs have lower *BER* compared with other schemes as shown in Fig. 3(b). Traditional PO algorithm without Gray-like mapping, it can reduce the *BER* to some extent. PAS reduces the probability of the outer high-power constellation points, thus improves the transmission system performance and weakens the nonlinearity influence. The 64QAM constellation shaped after PAS is shown in the inset of Fig. 3(b). Extensive simulation is tried out to obtain the optimal solution for the optimized 64QAM constellation by NNs. The simulation results show that the NNs scheme can obtain > 0.2dB gain higher than that of conventional 64QAM as shown in the inset of Fig. 3(c).

4. Conclusion

This paper introduces a NNs-based solution to the problem of geometric constellation optimization, which can operate over a wide range of SNRs and is not limited to specific channel model. It is shown that NNs-based GS scheme outperforms unshaped QAM in terms of *GMI*, and nearly reaches the capacity on an AWGN channel. The presented results are promising for other channel models, left as future research directions.

5. References

- [1] Chen B, Okonkwo C, et al, "Increasing achievable information rates via geometric shaping," ECOC, 1-3 (2018).
- [2] Zhang S, Yaman F, et al, "A generalized pairwise optimization for designing multi-dimensional modulation formats," OFC, 1-3 (2017).
- [3] Gümüş K, Alvarado A, et al, "End-to-end learning of geometrical shaping maximizing generalized mutual information," OFC, 1-3 (2020).
- Böcherer G, Steiner F, et al, "Bandwidth efficient and rate-matched low-density parity-check coded modulation," IEEE Transactions on communications, 63(12), 4651-4665(2015).
- [5] Pilori D, Nespola A, et al "Non-linear phase noise mitigation over systems using constellation shaping," JLT, 37(14), 3457-3482 (2019).